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A new approach to H_∞ filtering for linear time-delay systems

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Abstract

This paper proposes a class of H_∞ filter design for continue-time systems with time-varying delay. Firstly, by exploiting a new Lyapunov function and using the convexity property of the matrix inequality, some delay-dependent stability conditions can be obtained for the asymptotical stability of the filtering-error system, which can lead to much less conservative analysis results. Secondly, based on the obtained conditions, the filter parameter matrixes can be obtained in terms of linear matrix inequalities (LMIs). Finally, two examples are given to demonstrate the effectiveness and the merit of the proposed method.

1. Introduction

The H_∞ filtering problem has been extensively discussed over the past decades and its applications in a variety of areas such as signal processing, signal estimation, pattern recognition, communications, control application and many practical control systems have been studied. The problem of filtering can be briefly described as the design of an estimator from the measured output to estimate the state of the given systems. One of its main advantages is that it is insensitive to the exact knowledge of the statistics of the noise

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signals. During the last few decades, the H_∞ filtering technique introduced in [1] has received increasing attention, for example [2–10] and the references therein. A number of useful results have appeared for analyzing the H_∞ filtering criterion without a time-varying delay [11–13]. However, time delays are frequently encountered in many practical engineering systems, for example, chemical, electronics, hydraulic or process control systems and networked control systems [14,15]. Recently, considerable researchers focus on studying various systems with time-varying delays, for examples, T–S fuzzy systems [16,17], Markovian jump systems [18,10], singular systems [19,20], neutral systems [21], stochastic systems [22,23], switched systems [24,25], and uncertain case of the above systems, etc.

Recently, the problem of H_∞ filtering of linear/nonlinear time-delay systems has also received much attention due to the fact that for many practical filtering applications, time-delays cannot be neglected in the procedure of filter design and their existence usually results in a poor performance [26–28]. Some nice results on H_∞ filtering for time-delay systems have been reported in the literature and there are two kinds of results, namely delay-independent filtering [29] and delay-dependent [9,10,30–33]. The delay-dependent results are usually less conservative, especially when the time-delay is small. The main objective of the delay-dependent H_∞ filtering is to obtain a filter such that the filtering error system allows a maximum delay bound for a fixed H_∞ performance or achieves a minimum H_∞ performance for a given delay bound.

This paper addressed the problem of H_∞ filter design for time delay systems with interval time-varying delay. The restriction on the derivative of the interval time-varying delay is removed, which means that a fast interval time-varying delay is allowed [34–36]. Compared to the existing methods, the main features of our method can be highlighted as:

1. A new Lyapunov function is constructed, which includes the lower and upper delay bound of interval time-varying delay.
2. Jessen's inequality (Lemma 1) and Projection theorem (Lemma 3) are employed in the derivation of our results.
3. Convexity of the matrix function (Lemma 2) for cross terms is employed to reduce the conservation.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n\times m$ real matrices, the superscript "T" stands for matrix transposition, I is the identity matrix of appropriate dimension. $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. The notation $X>0$ (respectively, $X\geq 0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). For a matrix B and two symmetric matrices A and C, $\left[\begin{smallmatrix} A & * \\ B & C \end{smallmatrix}\right]$ denote a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. Systems description and preliminaries

Consider the following linear systems with time-varying delay:

$$\begin{cases} \dot{x}(t) = A_0x(t) + A_1x(t-\tau(t)) + A_w\omega(t) \\ y(t) = C_0x(t) + C_1x(t-\tau(t)) + C_w\omega(t) \\ z(t) = L_0x(t) + L_1x(t-\tau(t)) + L_w\omega(t) \\ x(\theta) = \phi(\theta), \quad \forall\theta\in[-\tau_2,-\tau_1] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^r$ is the measurement vector, $\omega(t) \in L_2[0,\infty)$ is the exogenous disturbance signal, $z(t) \in \mathbb{R}^p$ is the signal to be estimated, A_0, A_1, A_ω, C_0, C_1, C_ω, L_0, L_1 and L_ω are constant matrices with appropriate dimensions, time delay $\tau(t)$ is a time-varying continuous function satisfying the following assumption:

$$0 \le \tau_1 \le \tau(t) \le \tau_2 < \infty, \quad \forall t > 0 \tag{2}$$

where τ_1 is the lower bound and τ_2 is the upper bound of the communication delay.

In this paper, the aim is to design a stable and order n_f linear filter of the state-space representation

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f y(t) \\ z_f(t) = C_f x_f(t) + D_f y(t) \end{cases} \tag{3}$$

where A_f, B_f, C_f, D_f are filter parameters to be determined.

Let $e(t) = \left[\begin{smallmatrix} x(t) \\ x_f(t) \end{smallmatrix}\right]$ and $\tilde{z}(t) = z(t) - z_f(t)$. Then we have the following filtering error system:

$$\begin{cases} \dot{e}(t) = \hat{A}_0 e(t) + \hat{A}_1 E e(t-\tau(t)) + \hat{A}_w \omega(t) \\ \tilde{z}(t) = \hat{L}_0 e(t) + \hat{L}_1 E e(t-\tau(t)) + \hat{L}_w \omega(t) \\ e(\theta) = [\phi^T(\theta), 0]^T, \quad \forall \theta \in [-\tau_2, -\tau_1] \end{cases} \tag{4}$$

where $E = [I_n \ 0]$ and

$$\hat{A}_0 = E^T A_0 E + HKB$$

$$\hat{A}_1 = E^T A_1 + HKFC_1$$

$$\hat{A}_\omega = E^T A_\omega + HKFC_\omega$$

$$\hat{L}_0 = L_0 E + DKB$$

$$\hat{L}_1 = L_1 + DKFC_1$$

$$\hat{L}_\omega = L_\omega + DKFC_\omega$$

in which $D = [-I_P \ 0]$ and

$$K = \begin{bmatrix} D_f & C_f \\ B_f & A_f \end{bmatrix}, \quad B = \begin{bmatrix} C_0 & 0 \\ 0 & I_{n_f} \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 0 \\ 0 & I_{n_f} \end{bmatrix}, \quad F = \begin{bmatrix} I_r \\ 0 \end{bmatrix}$$

The H_∞ filtering problem addressed in this paper is to design a filter of form (3) such that

- The filtering error systems (4) with $\omega(t) = 0$ is asymptotically stable.
- The H_∞ performance $\|\tilde{z}(t)\|_2 < \gamma \|\omega(t)\|_2$ is guaranteed for all nonzero $\omega(t) \in L_2[0,\infty)$ and a prescribed $\gamma > 0$ under the condition $e(\theta) = 0$, $\forall \theta \in [-\tau_2, -\tau_1]$.

The following lemmas are needed in the proof of our main results.

Lemma 1 (*Jessen's inequality, Gu et al.* [37]). *For any constant matrix* $R \in \mathbb{R}^{n \times n}$, $R = R^T > 0$, *constant* $\tau_1 > 0$ *and vector function* $\dot{x} : [-\tau_1, 0] \to \mathbb{R}^n$ *such that the following* *integration is well defined, it holds that*

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds \le \begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R & * \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix} \tag{5}$$

Lemma 2 (*Convexity of the matrix function, Yue et al. [38]*). *Suppose* $0 \le \tau_m \le \tau(t) \le \tau_M$, Ξ_1, Ξ_2 *and* Ω *are constant matrices of appropriate dimensions, then*

$$(\tau(t)-\tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \tag{6}$$

if and only if

$$(\tau_M - \tau_m)\Xi_1 + \Omega < 0$$

and

$$(\tau_M - \tau_m)\Xi_2 + \Omega < 0$$

hold.

Lemma 3 (*Projection theorem, Gahinet and Apkarian [39]*). *Consider a symmetric matrix* $\Xi \in \mathbb{R}^{n \times n}$ *and two matrices* Π *and* Γ *with column dimension n. Then there exist a matrix* Θ *of compatible dimensions such that*

$$\Xi + \Pi^T \Theta \Gamma + \Gamma^T \Theta^T \Pi < 0 \tag{7}$$

if and only if

$$(\Pi_\perp)^T \Xi \Pi_\perp < 0$$

and

$$(\Gamma_\perp)^T \Xi \Gamma_\perp < 0$$

hold, where $\Pi_\perp$ *and* $\Gamma_\perp$ *denote the orthogonal complements of* Π *and* Γ; *respectively.*

3. Main results

Theorem 1. *For some given constants* $0 \le \tau_1 \le \tau_2$ *and* γ, *the augmented systems* (4) *are asymptotically stable with a prescribed* H_∞ *performance* γ *if there exist* $P>0$, $Q_1>0$, $Q_2>0$, $R_1>0$, $R_2>0$ *and* M_i, N_i $(i=1,2\ldots5)$ *with appropriate dimensions such that*

$$\Psi = \begin{bmatrix} \Xi_{11} & * & * \\ \Xi_{21} & \Xi_{22} & * \\ \Xi_{31}(s) & 0 & -R_2 \end{bmatrix} < 0, \quad s=1,2 \tag{8}$$

where

$$\Xi_{11} = \begin{bmatrix} \Upsilon_1 & * & * & * & * \\ R_1 E + N_1^T & \Upsilon_2 & * & * & * \\ \hat{A}_1^T P - N_1^T + M_1^T & N_3 - N_2^T + M_2^T & \Upsilon_3 & * & * \\ -M_1^T & N_4 - M_2^T & -N_4 + M_4 - M_3^T & -Q_2 - M_4 - M_4^T & * \\ \hat{A}_w^T P & N_5 & -N_5 + M_5 & -M_5 & -\gamma^2 I \end{bmatrix}$$

$$\Xi_{21}=\begin{bmatrix}\hat{L}_0 & 0 & \hat{L}_1 & 0 & \hat{L}_w\\ \tau_1 R_1 E\hat{A}_0 & 0 & \tau_1 R_1 E\hat{A}_1 & 0 & \tau_1 R_1 E\hat{A}_w\\ \sqrt{\tau_{21}}R_2 E\hat{A}_0 & 0 & \sqrt{\tau_{21}}R_2 E\hat{A}_1 & 0 & \sqrt{\tau_{21}}R_2 E\hat{A}_w\end{bmatrix}$$

$$\Xi_{22}=\mathrm{diag}\{-I,-R_1,-R_2\}$$

$$\Xi_{31}(1)=[\sqrt{\tau_{21}}N_1^T\ \ \sqrt{\tau_{21}}N_2^T\ \ \sqrt{\tau_{21}}N_3^T\ \ \sqrt{\tau_{21}}N_4^T\ \ \sqrt{\tau_{21}}N_5^T]$$

$$\Xi_{31}(2)=[\sqrt{\tau_{21}}M_1^T\ \ \sqrt{\tau_{21}}M_2^T\ \ \sqrt{\tau_{21}}M_3^T\ \ \sqrt{\tau_{21}}M_4^T\ \ \sqrt{\tau_{21}}M_5^T]$$

$$\Upsilon_1=P\hat{A}_0+\hat{A}_0^TP+E^T(Q_1+Q_2-R_1)E$$

$$\Upsilon_2=-Q_1-R_1+N_2+N_2^T$$

$$\Upsilon_3=M_3+M_3^T-N_3-N_3^T$$

$$\tau_{21}=\tau_2-\tau_1$$

Proof. Construct a Lyapunov functional candidate as

$$V(t,e_t)=V_1(t,e_t)+V_2(t,e_t)+V_3(t,e_t) \tag{9}$$

where

$$V_1(t,e_t)=e^T(t)Pe(t)$$

$$V_2(t,e_t)=\int_{t-\tau_1}^{t}x^T(t)Q_1x(t)\,dt+\int_{t-\tau_2}^{t}x^T(t)Q_2x(t)\,dt$$

$$V_3(t,e_t)=\tau_1\int_{-\tau_1}^{0}\int_{t+s}^{t}\dot{x}^T(v)R_1\dot{x}(v)\,dv\,ds+\int_{t-\tau_2}^{t-\tau_1}\int_{s}^{t}\dot{x}^T(v)R_2\dot{x}(v)\,dv\,ds$$

and P,Q_1,Q_2,R_1,R_2,R_3 are to be determined. Taking the time derivative of $V(t,e_t)$ with respect to t along the trajectory of Eq. (4) yields, we have

$$\dot{v}_1(t,e_t)=2e^T(t)P\dot{e}(t)=2e^T(t)p[\hat{A}_0e(t)+\hat{A}_1Ee(t-\tau(t))+\hat{A}_w\omega(t)] \tag{10}$$

$$\begin{aligned}\dot{v}_2(t,e_t)&=x^T(t)(Q_1+Q_2)x(t)-x^T(t-\tau_1)Q_1x(t-\tau_1)-x^T(t-\tau_2)Q_2x(t-\tau_2)\\&=e^T(t)[E^T(Q_1+Q_2)E]e(t)-x^T(t-\tau_1)Q_1x(t-\tau_1)-x^T(t-\tau_2)Q_2x(t-\tau_2)\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{v}_3(t,e_t)=&\tau_1^2\dot{e}^T(t)E^TR_1E\dot{e}(t)-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\\&+\tau_{21}\dot{e}^T(t)E^TR_2E\dot{e}(t)-\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)\,ds\end{aligned} \tag{12}$$

Applying Lemma 1 (Jessen's inequality), we have

$$-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)\,ds\le\begin{bmatrix}e(t)\\x(t-\tau_1)\end{bmatrix}^T\begin{bmatrix}-E^TR_1E & E^TR_1\\R_1E & -R_1\end{bmatrix}\begin{bmatrix}e(t)\\x(t-\tau_1)\end{bmatrix} \tag{13}$$

Employing the free-weighting matrices method, we have

$$2\zeta^T(t)N\left[x(t-\tau_1)-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\,ds\right]=0 \tag{14}$$

$$2\zeta^T(t)M\left[x(t-\tau(t))-x(t-\tau_2)-\int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\,ds\right]=0 \tag{15}$$

where

$$\zeta^T(t)=[e^T(t)\;\; x^T(t-\tau_1)\;\; x^T(t-\tau(t))\;\; x^T(t-\tau_2)\;\; \omega^T(t)]$$

$$N^T=[N_1^T\;\; N_2^T\;\; N_3^T\;\; N_4^T\;\; N_5^T]$$

$$M^T=[M_1^T\;\; M_2^T\;\; M_3^T\;\; M_4^T\;\; M_5^T]$$

There exists R_2 such that

$$-2\zeta^T(t)N\int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\,ds\leq\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)\,ds+(\tau(t)-\tau_1)\zeta^T(t)NR_2^{-1}N^T\zeta(t) \tag{16}$$

$$-2\zeta^T(t)M\int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\,ds\leq\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)R_2\dot{x}(s)\,ds+(\tau_2-\tau(t))\zeta^T(t)MR_2^{-1}M^T\zeta(t) \tag{17}$$

Adding Eqs. (14) and (15) to the right side of $\dot{V}(t,e_t)$ and substituting Eqs. (13), (16), (17) into it, we have

$$\begin{aligned}\dot{V}(t,e_t)\leq{}&2e^T(t)P[\hat{A}_0e(t)+\hat{A}_1Ee(t-\tau(t))+\hat{A}_w\omega(t)]+e^T(t)[E^T(Q_1+Q_2)E]e(t)\\&-x^T(t-\tau_1)Q_1x(t-\tau_1)-x^T(t-\tau_2)Q_2x(t-\tau_2)+\tau_1^2\dot{e}^T(t)E^TR_1E\dot{e}(t)\\&+\tau_{21}\dot{e}^T(t)E^TR_2E\dot{e}(t)+\begin{bmatrix}e(t)\\x(t-\tau_1)\end{bmatrix}^T\begin{bmatrix}-E^TR_1E & E^TR_1\\R_1E & -R_1\end{bmatrix}\begin{bmatrix}e(t)\\x(t-\tau_1)\end{bmatrix}\\&+2\zeta^T(t)N[x(t-\tau_1)-x(t-\tau(t))]+2\zeta^T(t)M[x(t-\tau(t))-x(t-\tau_2)]\\&+(\tau(t)-\tau_1)\zeta^T(t)NR_2^{-1}N^T\zeta(t)+(\tau_2-\tau(t))\zeta^T(t)MR_2^{-1}M^T\zeta(t)\end{aligned} \tag{18}$$

By Schur complement, from Eq. (18), we can obtain

$$\begin{aligned}&\dot{V}(t,e_t)-\gamma^2w^T(t)w(t)+\tilde{z}^T(t)\tilde{z}(t)\\&\quad\leq\zeta^T(t)\Xi_{11}\zeta(t)+\tau_1^2\dot{e}^T(t)E^TR_1E\dot{e}(t)+\tau_{21}\dot{e}^T(t)E^TR_2E+\tilde{z}^T(t)\tilde{z}(t)\\&\qquad+(\tau(t)-\tau_1)\zeta^T(t)NR_2^{-1}N^T\zeta(t)+(\tau_2-\tau(t))\zeta^T(t)MR_2^{-1}M^T\zeta(t)\end{aligned} \tag{19}$$

By using Lemma 2 (Convexity of the matrix function) and Schur complement, from Eq. (19), it is easy to see that Eq. (8) with $s=1$, 2 can lead to

$$\dot{V}(t,e_t)\leq-\tilde{z}^T(t)\tilde{z}(t)+\gamma^2\omega^T(t)\omega(t) \tag{20}$$

Under zero initial condition, integrating both sides of Eq. (20) from t_0 to t and letting $t\to\infty$, we have $\|\tilde{z}(t)\|_2\leq\gamma\|\omega(t)\|_2$.

Next, we consider the asymptotical stability of the systems (4). When $\omega(t)=0$, combining Eqs. (8) and (20) together, we have $\dot{V}(t,e_t)<0$, which gives $\dot{V}(t,e_t)<-\rho\|x(t)\|^2$ for a sufficiently small $\rho>0$, and ensures the asymptotical stability of the systems (4) for any delay satisfying Eq. (2). This completes the proof. □

Remark 1. Using the methods in [40,41], the time-varying delay $\tau(t)$ often appears in the derivation of the Lyapunov functional or the introduced free weighing matrix equations, such as $\int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)\,ds$ and $\tau(t)\zeta^T(t)X\zeta(t)$ ($R>0$ and $X>0$), which is enlarged to $\int_{t-\tau_2}^{t} \dot{x}^T(s)R\dot{x}(s)\,ds$ and $\tau_2\zeta^T(t)X\zeta(t)$ by using the method in [40,41], then the estimation errors $\int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)\,ds$ and $(\tau_2-\tau(t))\zeta^T(t)X\zeta(t)$ are ignored, which will unavoidably lead to some degree of conservativeness. However, from the proof of Theorem 1, it can be seen that there is no enlargement for $\tau(t)$ by using Lemma 2 (Convexity of the matrix function), therefore the conservatism caused by enlarging $\tau(t)$ to τ_2 can be avoided.

In order to show the reduced conservatism of our stability criteria, we consider the following systems as a special case of Eq. (1)

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau(t)) \\ x(t) = \phi(t), \quad t \in [-\tau_2, 0] \end{cases} \tag{21}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $\phi(t)$ is a continuously differentiable vector-valued function; $A \in \mathbb{R}^{n\times n}$, $A_d \in \mathbb{R}^{n\times n}$ are constant system matrices; $\tau(t)$ is a time-varying continuous function satisfying Eq. (2).

Using the same method in Theorem 1, we can get the following results.

Corollary 1. *Given scalars* $0<\tau_1<\tau_2$, *system* (21) *with a time-varying delay satisfying Eq.* (2) *is asymptotically stable if there exists matrices* $P>0$, $Q_1>0$, $Q_2>0$, $R_1>0$, $R_2>0$ *and* N, M *of appropriate dimensions such that the following LMIs hold*:

$$\begin{bmatrix} \Omega_{11} + \Gamma + \Gamma^T & * & * \\ \Omega_{21} & \Omega_{22} & * \\ \Omega_{31}(s) & 0 & -R_2 \end{bmatrix} < 0, \quad s = 1, 2 \tag{22}$$

where

$$\Omega_{11} = \begin{bmatrix} PA + A^T P + Q_1 + Q_2 - R_1 & * & * & * \\ R_1 & -Q_1 - R_1 & * & * \\ A_d^T P & 0 & 0 & * \\ 0 & 0 & 0 & -Q_2 \end{bmatrix}$$

$$\Gamma = [0 \;\; N \;\; -N+M \;\; -M]$$

$$\Omega_{21} = \begin{bmatrix} \tau_1 R_1 A & 0 & \tau_1 R_1 A_d & 0 \\ \sqrt{\tau_{21}} R_2 A & 0 & \sqrt{\tau_{21}} R_2 A_d & 0 \end{bmatrix}$$

$$\Omega_{22} = \mathrm{diag}\{-R_1, -R_2\}$$

$$\Omega_{31}(1) = \sqrt{\tau_{21}} N^T, \quad \Omega_{31}(2) = \sqrt{\tau_{21}} M^T$$

Proof. Choose the Lyapunov functional as

$$V(x_t) = x^T(t)Px(t) + \int_{t-\tau_1}^{t} x^T(t)Q_1 x(t)\,dt + \int_{t-\tau_2}^{t} x^T(t)Q_2 x(t)\,dt$$

$$+\tau_1\int_{-\tau_1}^{0}\int_{t+s}^{t}\dot{x}^T(v)R_1\dot{x}(v)\,dv\,ds+\int_{t-\tau_2}^{t-\tau_1}\int_{s}^{t}\dot{x}^T(v)R_2\dot{x}(v)\,dv\,ds \tag{23}$$

and the free weighting matrix as

$$2\zeta^T(t)N\left[x(t-\tau_1)-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\,ds\right]=0 \tag{24}$$

$$2\zeta^T(t)M\left[x(t-\tau(t))-x(t-\tau_2)-\int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\,ds\right]=0 \tag{25}$$

where

$$\zeta^T(t)=[e^T(t)\ \ x^T(t-\tau_1)\ \ x^T(t-\tau(t))\ \ x^T(t-\tau_2)]$$

$$N^T=[N_1^T\ \ N_2^T\ \ N_3^T\ \ N_4^T]$$

$$M^T=[M_1^T\ \ M_2^T\ \ M_3^T\ \ M_4^T]$$

Then, similar to the proof of Theorem 1, Eq. (22) can be obtained, hence omitted. □

Remark 2. From Corollary 1, we can obtain the admissible upper bounds τ_2 of the time delay through solving the following maximum problem by using LMI SOLVER FEASP in MATLAB LMI tool box [42].

$$\begin{aligned}&\max\quad \tau_2\\ &\quad \text{subject to}\quad \text{LMI (22)}\end{aligned}$$

In the following, we are seeking to design the H_∞ filtering based on Theorem 1.

Theorem 2. *Let* $[W_1^T\ W_2^T\ W_3^T]^T$ *be the orthogonal complement of* $[C_0\ C_1\ C_w]^T$. *For some given constants* $0\le\tau_1\le\tau_2$ *and* γ, *the* H_∞ *filtering problem for system* (1) *is solvable if there exist* $\tilde{X}>0$, $Y>0$, $Q_1>0$, $Q_2>0$, $R_1>0$, $R_2>0$ *and* N_{10}, M_{10}, $\hat{N}_{11}$, $\hat{M}_{11}$, M_i,N_i $(i=2,3,\ldots,5)$ *of appropriate dimensions such that the following LMIs hold*:

$$\begin{bmatrix}\Omega_{11} & * & *\\ \Omega_{21} & \Omega_{22} & *\\ \Omega_{31}(s) & 0 & -R_2\end{bmatrix}<0,\quad s=1,2 \tag{26}$$

$$\begin{bmatrix}\Pi_{11} & * & *\\ \Pi_{21} & \Pi_{22} & *\\ \Pi_{31}(s) & 0 & -R_2\end{bmatrix}<0,\quad s=1,2 \tag{27}$$

$$\begin{bmatrix}\tilde{X} & *\\ \tilde{X} & Y\end{bmatrix}\ge 0 \tag{28}$$

where

$$\Omega_{11}=\begin{bmatrix}\Gamma_1 & * & * \\ \Gamma_2 & -Q_1-R_1+N_2+N_2^T & * \\ \Gamma_3 & N_4-M_2^T & -Q_2-M_4-M_4^T\end{bmatrix}$$

$$\Omega_{21}=\begin{bmatrix}L_0W_1+L_1W_2+L_wW_3 & 0 & 0 \\ \Gamma_4 & 0 & 0 \\ \Gamma_5 & 0 & 0\end{bmatrix}$$

$$\Omega_{22}=\mathrm{diag}\{-I,-R_1,-R_2\}$$

$$\Omega_{31}(1)=[\Gamma_6\ \ \sqrt{\tau_{21}}N_2^T\ \ \sqrt{\tau_{21}}N_4^T],\quad \Omega_{31}(2)=[\Gamma_7\ \ \sqrt{\tau_{21}}M_2^T\ \ \sqrt{\tau_{21}}M_4^T]$$

$$\begin{aligned}\Gamma_1 ={}& W_1^TYA_0W_1+W_1^TA_0^TYW_1+W_1^T(Q_1+Q_2-R_1)W_1+W_1^TYA_1W_2\\ &+W_2^TA_1^TYW_1-W_1^TN_{10}W_2-W_2^TN_{10}^TW_1+W_1^TM_{10}W_2+W_2^TM_{10}^TW_1\\ &+W_1^TYA_wW_3+W_3^TA_w^TYW_1+W_2^TM_3W_2+W_2^TM_3^TW_2-W_2^TN_3W_2\\ &-W_2^TN_3^TW_2-W_3^TN_5W_2-W_2^TN_5^TW_3+W_3^TM_5W_2+W_2^TM_5^TW_3-\gamma^2W_3^TW_3\end{aligned}$$

$$\Gamma_2=R_1W_1+N_{10}^TW_1+N_3^TW_2-N_2W_2+M_2W_2+N_5^TW_3$$

$$\Gamma_3=-M_{10}^TW_1-N_4W_2+M_4W_2-M_2^TW_2-M_5^TW_3$$

$$\Gamma_4=\tau_1R_1A_0W_1+\tau_1R_1A_1W_2+\tau_1R_1A_wW_3$$

$$\Gamma_5=\sqrt{\tau_{21}}R_2A_0W_1+\sqrt{\tau_{21}}R_2A_1W_2+\sqrt{\tau_{21}}R_2A_wW_3$$

$$\Gamma_6=\sqrt{\tau_{21}}N_{10}^TW_1+\sqrt{\tau_{21}}N_3^TW_2+\sqrt{\tau_{21}}N_5^TW_3$$

$$\Gamma_7=\sqrt{\tau_{21}}M_{10}^TW_1+\sqrt{\tau_{21}}M_3^TW_2+\sqrt{\tau_{21}}M_5^TW_3$$

$$\Pi_{11}=\begin{bmatrix}\Lambda_1 & * & * & * & * \\ R_1+N_{10}^T+\hat{N}_{11}^T & \Lambda_2 & * & * & * \\ \Lambda_3 & N_3-N_2^T+M_2^T & \Lambda_4 & * & * \\ -M_{10}^T-\hat{M}_{11}^T & N_4-M_2^T & -N_4+M_4-M_3^T & -Q_2-M_4-M_4^T & * \\ \Lambda_5 & N_5 & -N_5+M_5 & -M_5 & -\gamma^2I\end{bmatrix}$$

$$\Pi_{21}=\begin{bmatrix}\tau_1R_1A_0 & 0 & \tau_1R_1A_1 & 0 & \tau_1R_1A_w \\ \sqrt{\tau_{21}}R_2A_0 & 0 & \sqrt{\tau_{21}}R_2A_1 & 0 & \sqrt{\tau_{21}}R_2A_w\end{bmatrix}$$

$$\Pi_{22}=\mathrm{diag}\{-R_1,-R_2\}$$

$$\Pi_{31}(1)=[\sqrt{\tau_{21}}N_{10}^T\ \ \sqrt{\tau_{21}}N_2^T\ \ \sqrt{\tau_{21}}N_3^T\ \ \sqrt{\tau_{21}}N_4^T\ \ \sqrt{\tau_{21}}N_5^T]$$

$$\Pi_{31}(2)=[\sqrt{\tau_{21}}M_{10}^T \ \ \sqrt{\tau_{21}}M_2^T \ \ \sqrt{\tau_{21}}M_3^T \ \ \sqrt{\tau_{21}}M_4^T \ \ \sqrt{\tau_{21}}M_5^T]$$

$$\Lambda_1 = YA_0 + A_0^T Y + Q_1 + Q_2 - R_1 + \tilde{X}A_0 + A_0^T\tilde{X} - YA_0 - A_0^T Y$$

$$\Lambda_2 = -Q_1 - R_1 + N_2 + N_2^T$$

$$\Lambda_3 = A_1^T Y - N_{10}^T + M_{10}^T + A_1^T\tilde{X} - A_1^T Y^T - \hat{N}_{11}^T + \hat{M}_{11}^T$$

$$\Lambda_4 = -N_3 - N_3^T + M_3 + M_3^T$$

$$\Lambda_5 = A_w^T Y + A_w^T\tilde{X} - A_w^T Y^T$$

Proof. Define

$$P=\begin{bmatrix} Y & \mathcal{N} \\ \mathcal{N}^T & \circledast \end{bmatrix}, \quad P^{-1}=\begin{bmatrix} X & \mathcal{M} \\ \mathcal{M}^T & \circledast \end{bmatrix} \tag{29}$$

where $X>0, Y>0 \in \mathbb{R}^{n\times n}$, and symbol $\circledast$ denotes the irrelevant part.

From Eq. (29), we have

$$XY + \mathcal{M}\mathcal{N}^T = I \tag{30}$$

From Eq. (30) and letting

$$N_1^T=[N_{10}^T \ \ N_{11}^T], \quad M_1^T=[M_{10}^T \ \ M_{11}^T]$$

then, rewrite Eq. (8) as

$$\Psi = \Psi_0 + \Sigma UK\Theta^T + (\Sigma UK\Theta^T)^T \tag{31}$$

where

$$\Psi_0=\begin{bmatrix} \Sigma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31}(s) & 0 & -R_2 \end{bmatrix}$$

$$\Sigma = \mathrm{diag}\{P,\underbrace{I,\ldots,I}_{8}\},$$

$$U=\begin{bmatrix} 0 & 0 \\ 0 & I \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -I & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Theta=\begin{bmatrix} C_0^T & 0 \\ 0 & I \\ 0 & 0 \\ C_1^T & 0 \\ 0 & 0 \\ C_\omega^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

in which

$$\Sigma_{11}=\begin{bmatrix} \Delta_1 & * & * & * & * & * \\ \mathcal{N}^TA_0 & 0 & * & * & * & * \\ R_1+N_{10}^T & N_{11}^T & \Delta_2 & * & * & * \\ A_1^TY-N_{10}^T+M_{10}^T & A_1^T\mathcal{N}-N_{11}^T+M_{11}^T & N_3-N_2^T+M_2^T & \Delta_3 & * & * \\ -M_{10}^T & -M_{11}^T & N_4-M_2^T & -N_4+M_4-M_3^T & \Delta_4 & * \\ A_w^TY & A_w^T\mathcal{N} & N_5 & -N_5+M_5 & -M_5 & -\gamma^2I \end{bmatrix}$$

$$\Sigma_{21}=\begin{bmatrix} L_0 & 0 & 0 & L_1 & 0 & L_w \\ \tau_1R_1A_0 & 0 & 0 & \tau_1R_1A_1 & 0 & \tau_1R_1A_w \\ \sqrt{\tau_{21}}R_2A_0 & 0 & 0 & \sqrt{\tau_{21}}R_2A_1 & 0 & \sqrt{\tau_{21}}R_2A_w \end{bmatrix}$$

$$\Sigma_{22}=\mathrm{diag}\{-I,-R_1,-R_2\}$$

$$\Sigma_{31}(1)=[\sqrt{\tau_{21}}N_{10}^T\ \ \sqrt{\tau_{21}}N_{11}^T\ \ \sqrt{\tau_{21}}N_2^T\ \ \sqrt{\tau_{21}}N_3^T\ \ \sqrt{\tau_{21}}N_4^T\ \ \sqrt{\tau_{21}}N_5^T]$$

$$\Sigma_{31}(2)=[\sqrt{\tau_{21}}M_{10}^T\ \ \sqrt{\tau_{21}}M_{11}^T\ \ \sqrt{\tau_{21}}M_2^T\ \ \sqrt{\tau_{21}}M_3^T\ \ \sqrt{\tau_{21}}M_4^T\ \ \sqrt{\tau_{21}}M_5^T]$$

$$\Delta_1=YA_0+A_0^TY+Q_1+Q_2-R_1,\quad \Delta_2=-Q_1-R_1+N_2+N_2^T$$

$$\Delta_3=-N_3-N_3^T+M_3+M_3^T,\quad \Delta_4=-Q_2-M_4-M_4^T$$

By using Lemma 2 (projection theorem), inequality (31) is solvable for some K if and only if

$$\Theta_\perp^T\Psi_0\Theta_\perp<0 \tag{32}$$

$$U_\perp^T\Sigma^{-1}\Psi_0\Sigma^{-1}U_\perp<0 \tag{33}$$

To simplify Eqs. (32) and (33), we can choose

$$\Theta_\perp=\begin{bmatrix} W_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ W_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ W_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},\quad U_\perp=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

After some simple algebraic manipulation, inequality (32) is equivalent to Eq. (26). Setting

$$\begin{cases} \tilde{X}=X^{-1} \\ \hat{M}_{11}=M_{11}^T\mathcal{M}^T\tilde{X} \\ \hat{N}_{11}=N_{11}^T\mathcal{M}^T\tilde{X} \end{cases}$$

and introducing $\prod = \mathrm{diag}\{\tilde{X}, \underbrace{I, \ldots, I}_{7}\}$, then, $\prod^T U_\perp^T \Sigma^{-1} \Psi_0 \Sigma^{-1} U_\perp \prod < 0$ is equivalent to Eq. (27).

Finally, using a method similar to that in [39], we can see that there exist a positive definite matrix P satisfying Eq. (29) if and only if $X - Y^{-1} \geq 0$, that is

$$X - Y^{-1} \geq 0 \;\Leftrightarrow\; \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \;\Leftrightarrow\; \begin{bmatrix} \tilde{X} & \tilde{X} \\ \tilde{X} & Y \end{bmatrix} \geq 0$$

which yields Eq. (28) . This completes the proof. □

Moreover, by using a method similar to that given in [8], the filter parameters $K = [\begin{smallmatrix} D_f & C_f \\ B_f & A_f \end{smallmatrix}]$ can be obtained as follows:

Remark 3. If LMIs (26)–(28) are feasible for matrix variables ($\tilde{X}$, Y, Q_1, Q_2, R_1, R_2), then the filter parameters $K = [\begin{smallmatrix} D_f & C_f \\ B_f & A_f \end{smallmatrix}]$ can be obtained by the following procedure:

① Compute two full-column rank matrixes $\mathcal{M}, \mathcal{N} \in \mathcal{R}^{n \times n_f}$, such that $\mathcal{M}\mathcal{N}^T = I - \tilde{X}^{-1} Y$.

② Calculate the matrix P by solving the matrix equation

$$\begin{bmatrix} Y & I \\ \mathcal{N}^T & 0 \end{bmatrix} = P \begin{bmatrix} I & \tilde{X}^{-1} \\ 0 & \mathcal{M}^T \end{bmatrix}. \tag{34}$$

③ Derive K by solving matrix inequality (8) with known matrices (P, R_1, R_2, R_3).

4. Example

In this section, two numerical examples are given to show the effectiveness and conservatism of our proposed results.

Example 1. Consider the system (21) with [43,44]

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}$$

For various τ_1, the admissible upper bounds τ_2 of the time delay are shown in Table 1.

As shown in Table 1, it is clear that the results obtained in this paper are less conservative than those in [43,44].

Table 1
Allowable upper bound of τ_2 for various τ_1.

Method	τ_1	0.3	0.5	0.8	1	2
[43]	τ_2	0.9431	1.0991	1.3476	1.5187	2.4000
[44]	τ_2	1.0715	1.2191	1.4539	1.6169	2.4798
Corollary 1	τ_2	1.2448	1.3806	1.6008	1.7555	2.5875

Example 2. Consider systems (1) with [8]

$$A_0=\begin{bmatrix}-2&0&0&0&0\\1&-0.9&0&0&0\\0&1&-2&0&0\\0&0&1&-0.9&0\\0&0&0&1&-1\end{bmatrix},\quad A_1=\begin{bmatrix}-1&0&0&0&0\\-1&-1&0&0&0\\0&-1&-1&0&0\\0&0&-1&-1&0\\0&0&0&-1&-1\end{bmatrix},\quad A_\omega=\begin{bmatrix}1\\1\\0\\1\\0\end{bmatrix}$$

$$C_0=\begin{bmatrix}1&1&-1&-1&1\\0&1&0&-1&-1\end{bmatrix},\quad C_1=\begin{bmatrix}-1&-1&1&1&1\\1&0&1&1&0\end{bmatrix},\quad C_\omega=\begin{bmatrix}1\\1\end{bmatrix}$$

$$L_0=\begin{bmatrix}0.5&0&0.5&0&0.5\\-0.5&0.5&0&0.5&0.5\\0.5&-0.5&0.5&0.5&0\end{bmatrix},\quad L_1=0,\quad L_\omega=0.$$

Let $\tau(t)\in[0.2,0.6],\gamma=8$, in terms of Remark 1, the corresponding filter parameters of Eq. (3) are obtained as

$$A_f=\begin{bmatrix}-2.5612&-0.2394&-0.3501&-1.3429&0.5941\\1.2096&-5.1486&-1.2477&-0.3368&0.0937\\-0.3014&-0.9563&-2.1545&-8.2770&-5.9241\\-1.8589&-0.6903&0.2398&-1.2430&1.5222\\3.7383&5.1382&4.2137&-14.0870&-14.5547\end{bmatrix}$$

$$B_f=\begin{bmatrix}-0.7235&-1.6114\\-7.4243&5.9016\\-5.5086&4.4904\\0.1553&-0.3160\\4.3165&-0.1600\end{bmatrix}$$

$$C_f=\begin{bmatrix}-0.0461&-0.1393&-0.1318&-0.2513&-0.2375\\-0.0714&-0.2112&-0.2036&-0.3943&-0.3796\\-0.0277&-0.0922&-0.0853&-0.1569&-0.1569\end{bmatrix}$$

$$D_f=\begin{bmatrix}0.0554&-0.0760\\0.0131&-0.0697\\-0.0318&-0.0759\end{bmatrix}$$

For this example, we calculated the achieved minimum H_∞ performances γ_{min} of the filtering-error system for various delays τ_2 when $\tau_1=0.2$ by using Theorem 2, and the calculated results are listed in Table 2.

With this filter, Fig. 1 shows the time-varying delay, Fig. 2 shows the state responses for the system (1) with the initial condition $\phi(t)=[-0.5\ -0.2\ 0\ 0.2\ 0.5]^T$ and the following

Table 2
The achieved H_∞ performances γ_{min} for various delay τ_2 when $\tau_1=0.2$.

Method	τ_2	0.3	0.5	0.8	1
[5]	γ_{min}	1.5121	1.7821	1.8763	1.9864
Theorem 2	γ_{min}	1.2315	1.3321	1.4369	1.6547

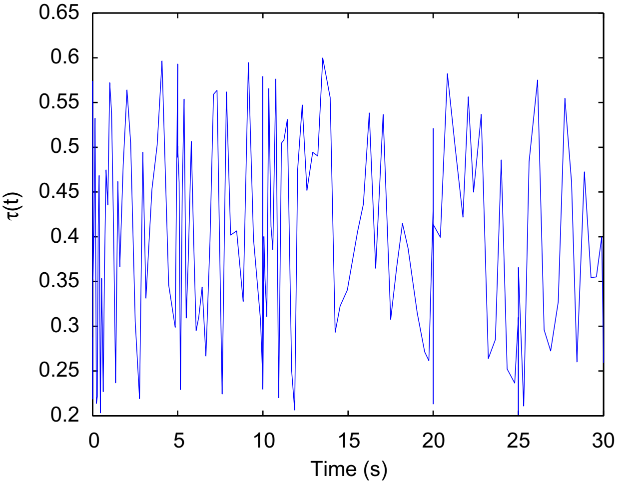

Fig. 1. Interval time-varying delay.

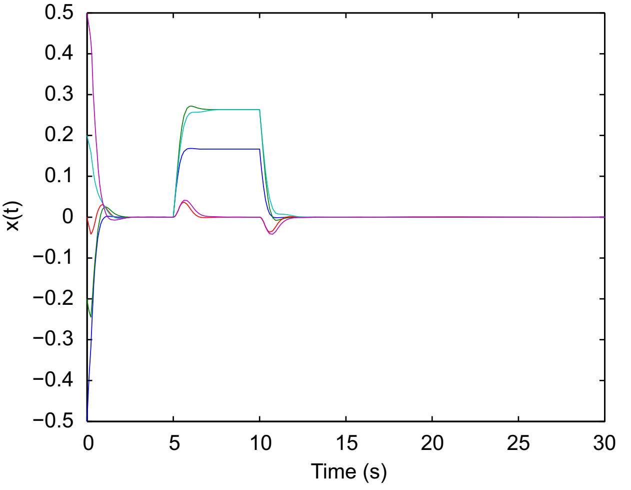

Fig. 2. State responses for the system (1).

exogenous disturbance input $w(t)$:

$$w(t)=\begin{cases}0.5, & 5s\le t\le 10s\\ 0 & \text{otherwise}\end{cases}$$

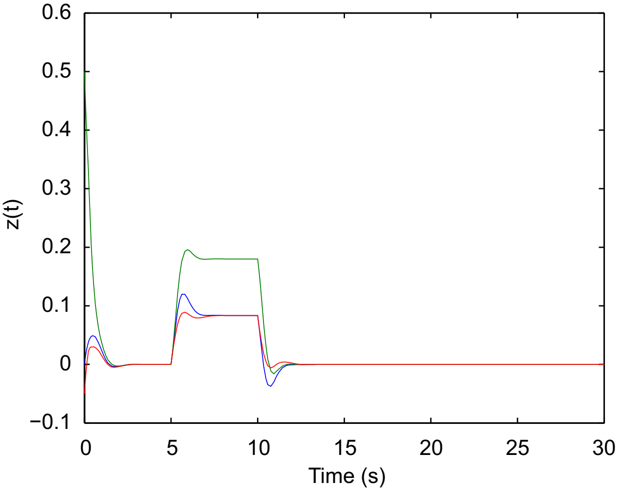

Fig. 3. Estimated signals for $z(t)$.

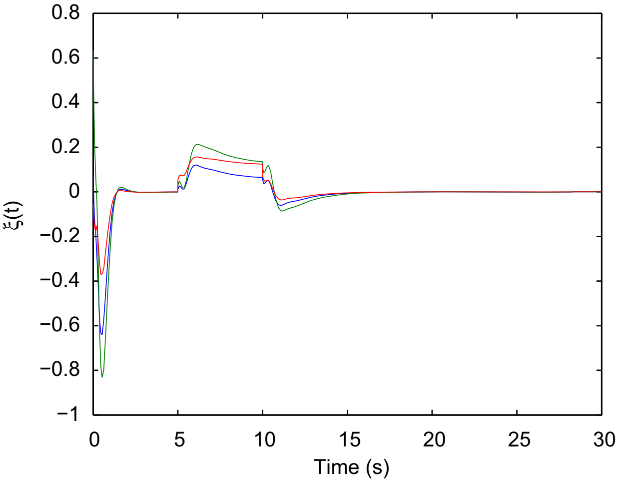

Fig. 4. Estimated signals with the designed filter.

Fig. 3 shows estimated signals for $z(t)$, Fig. 4 shows the error estimation signal of $\xi(t)=z(t)-z_f(t)$. It can be seen from Fig. 4 that using the designed filter of Eq. (3) to estimate the signal can achieve the good effect. As can be seen in Figs. 2–4, the effectiveness of the proposed method is apparent. It is also clear to see that the designed H_∞ filter has achieved well robustness with disturbance. It should be noted that the proposed approach is executed in the mode of off-line, which means that the computational burden of this method does not limit its applicability.

5. Conclusion

In this paper, we have studied the problem of H_∞ filter design for time delay systems with fast time-varying delay. A new method has been proposed to solve H_∞ filter design problem by employing convexity of the matrix function and projection theorem. With the proposed approach, an LMI-based sufficient condition for the existence of the desired H_∞ filter has been derived. Two examples have been carried out to demonstrate the effectiveness and the merit of the proposed method.

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